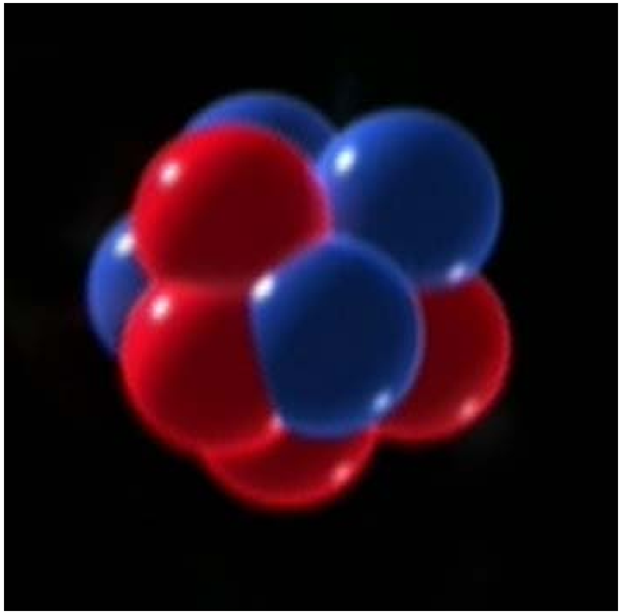
Introduction to Nuclei – II

(The physical properties)

“Whatever Nature has in store for mankind, unpleasant as it may be, men must accept, for ignorance is never better than knowledge”

- Enrico Fermi

Nuclear Composition



The atomic nucleus is made of N neutrons and Z protons

The number of nucleons, A = N + Z

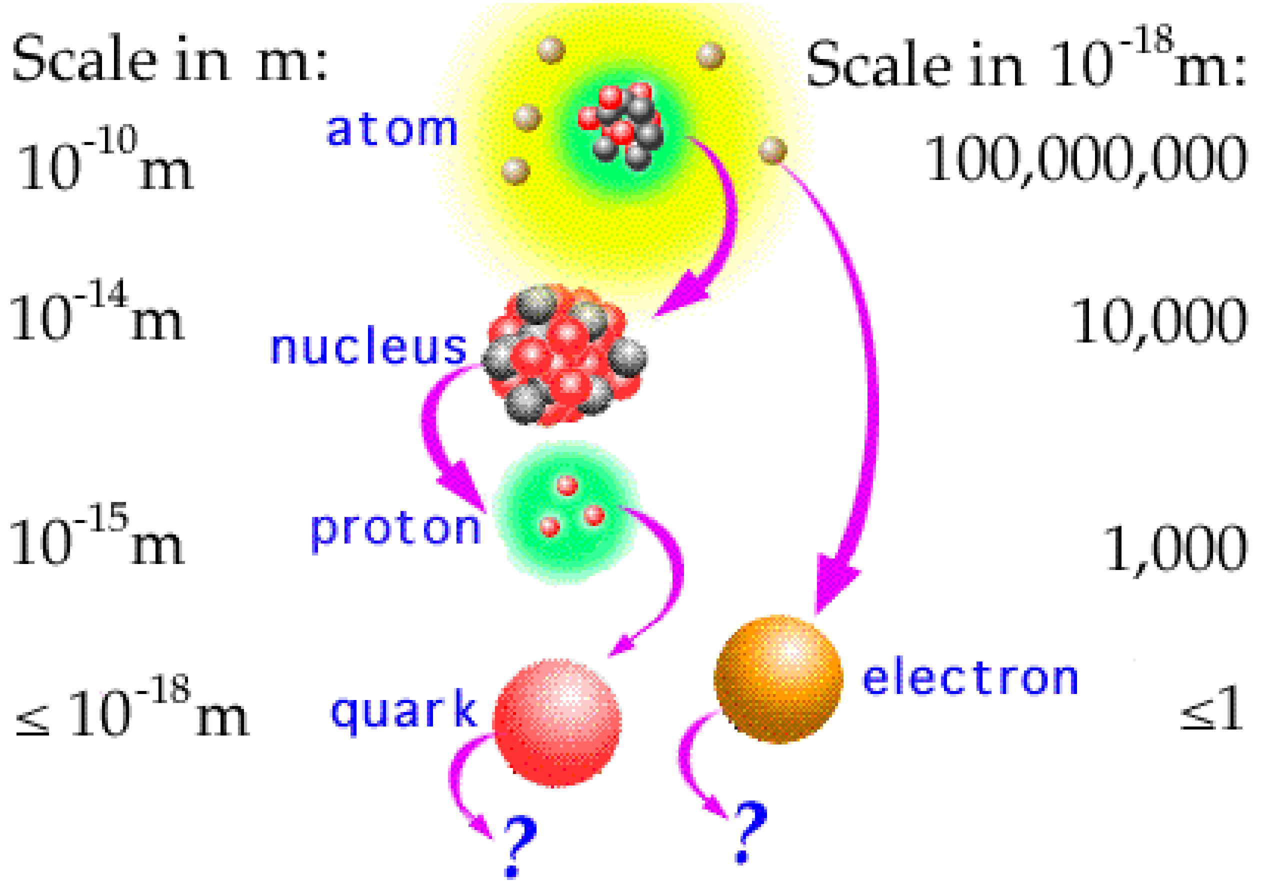
The general notation is, ${}^{A}_{Z}X_{N}$

particle	m (kg)	m (amu)	mc^2 (MeV)	charge	spin
proton	1.6727×10^{-27}	1.007276	938.27	+e	1/2
neutron	1.6749×10^{-27}	1.008665	939.57	0	1/2

Nuclear Size

Radius of a typical nucleus is about 10 fm = 10^{-14} m

Neutron scattering from nuclei can determine the nuclear radius.



radius	$R = (1.07 \pm 0.02) A^{1/3}$ **fm**

$$\mathbf{1\,fm = 10^{-15}\ m}$$

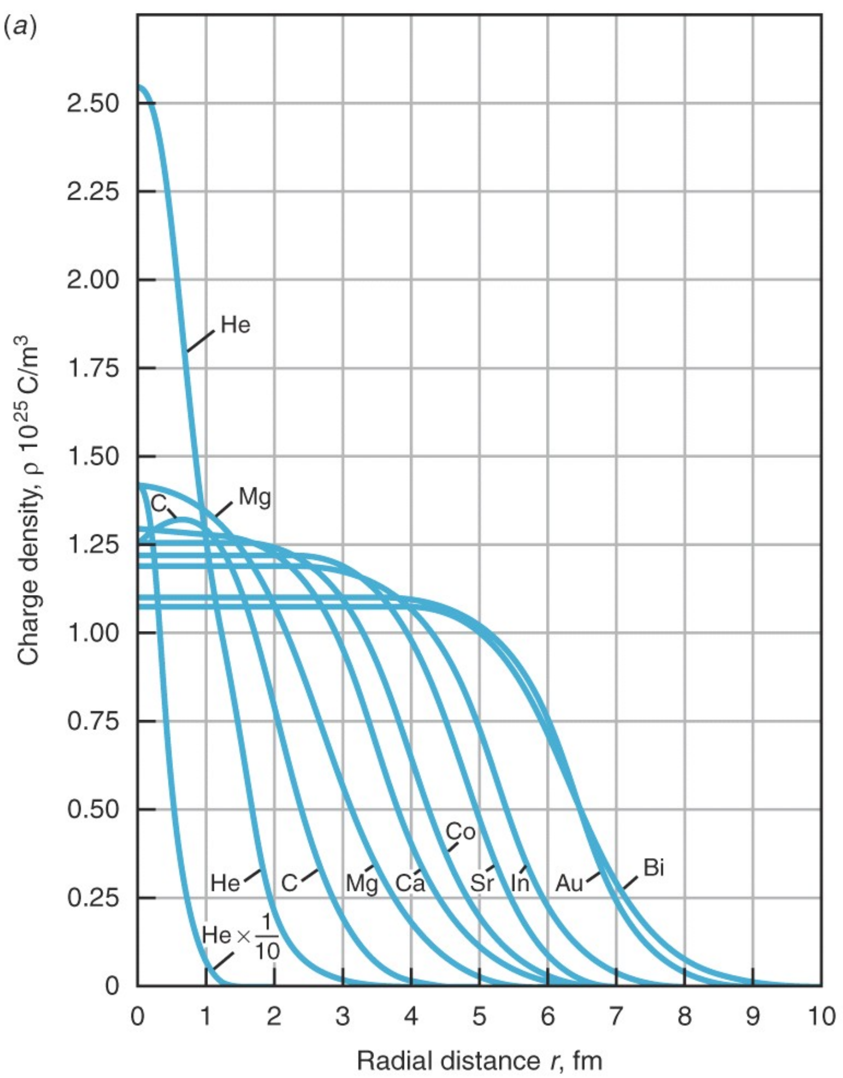
Nuclear Charge Distribution

The atomic nucleus is positively charged

In the interior of heavier nuclei (Au, Bi, …), charge is uniformly distributed.

For lighter nuclei (He, C, Mg ..) there is a steady decrease of density

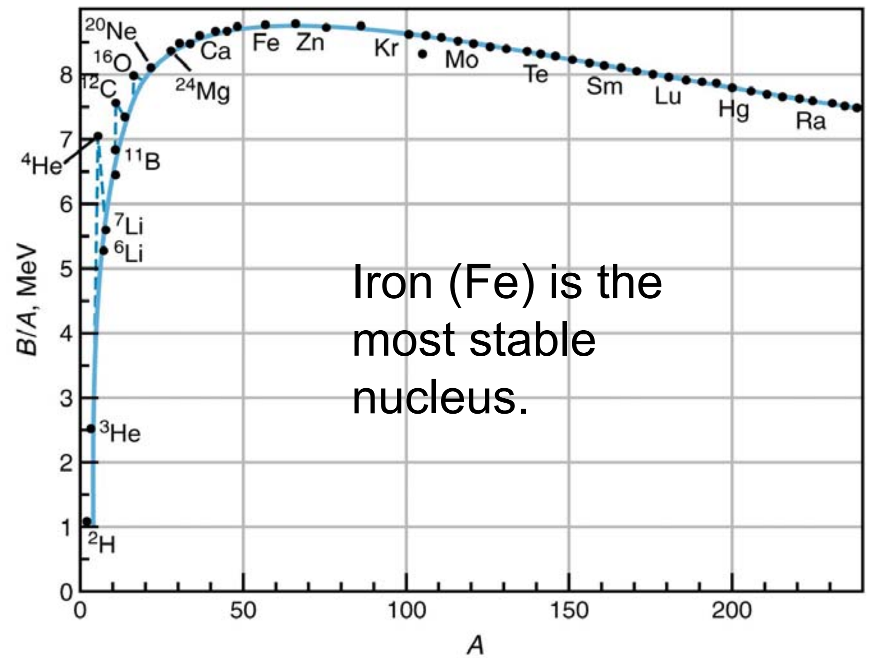
Elastic scattering of electrons from nuclei can accurately determine the nuclear charge distribution.



Nuclear Masses and Binding Energies

Binding Energy determines the stability of a nucleus

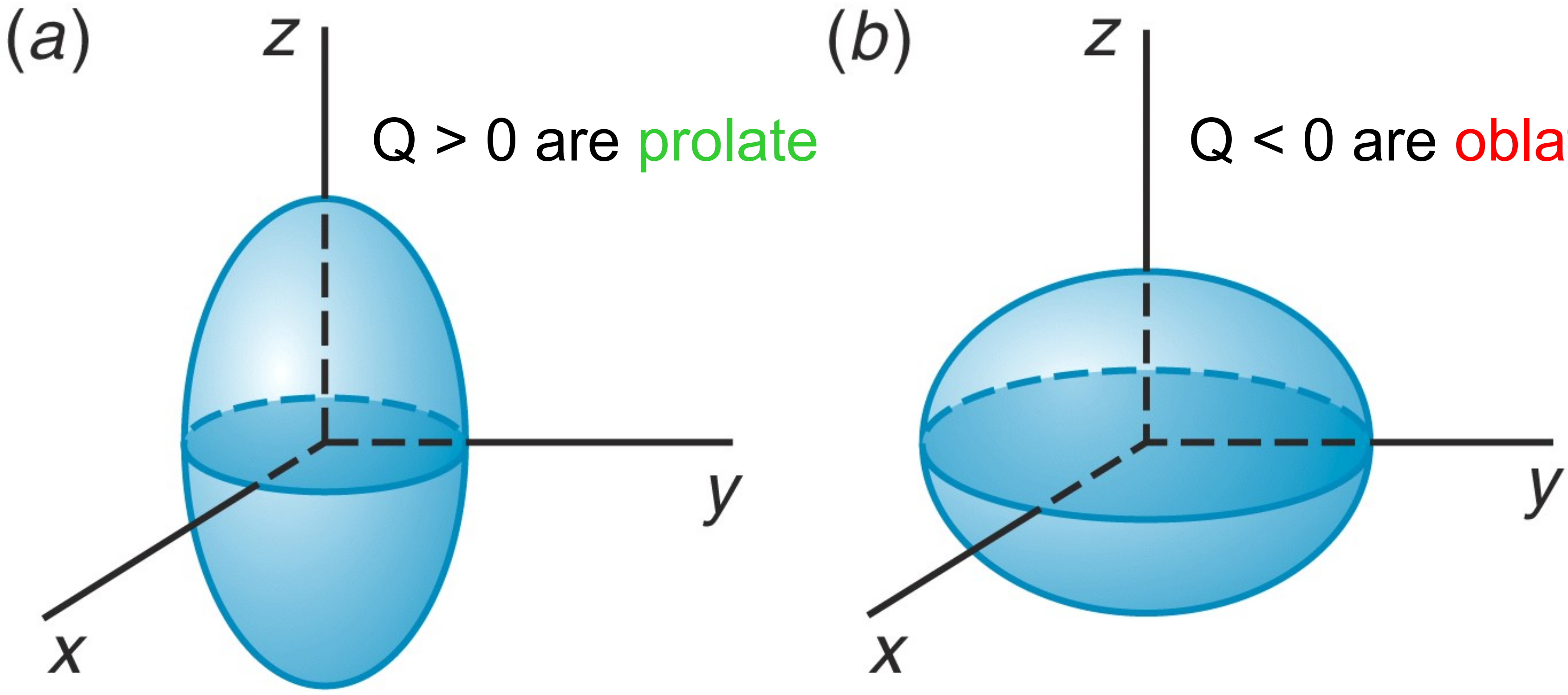
Binding Energy = sum of all proton and neutron mass-energies minus nuclear mass-energy



$$B = Zm_{proton}c^2 + Nm_{neutron}c^2 - M_{nucleus}c^2 > 0$$

For all but the lightest nuclei the average binding energy per nucleon is about 8 MeV.

Nuclear Shapes



Q > 0 are prolate

Q < 0 are oblate

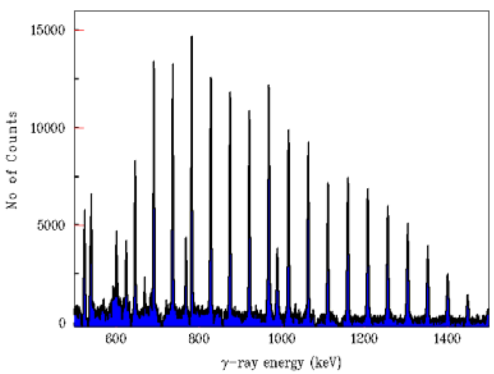
Nuclei with quadrupole Q = 0 are spherical.

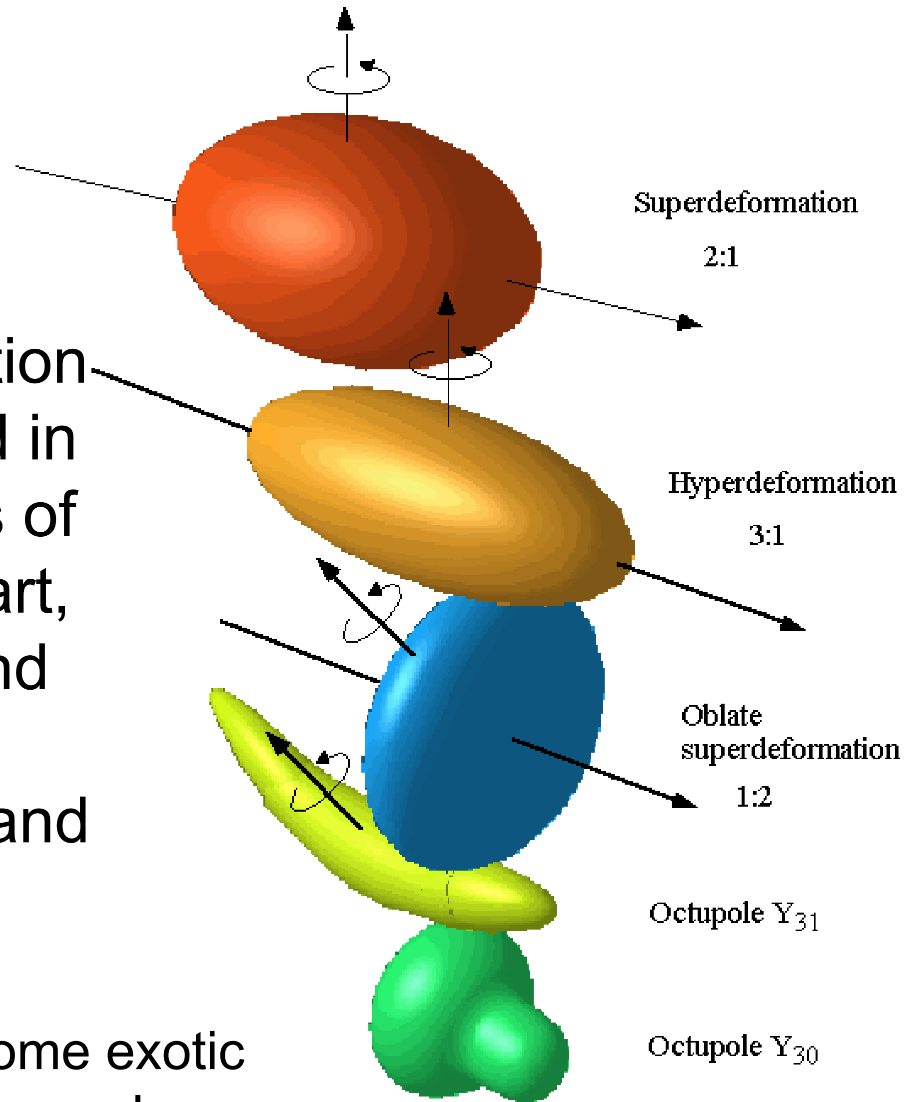
Electric quadrupole moment Q is a measure of the shape of a nucleus

Nuclear Rotations

A nucleus can rotate with very high spin and deform itself

Super-deformation has been found in several regions of the nuclear chart, in nuclei around A=60, A=80, A=130, A=150 and A=190.

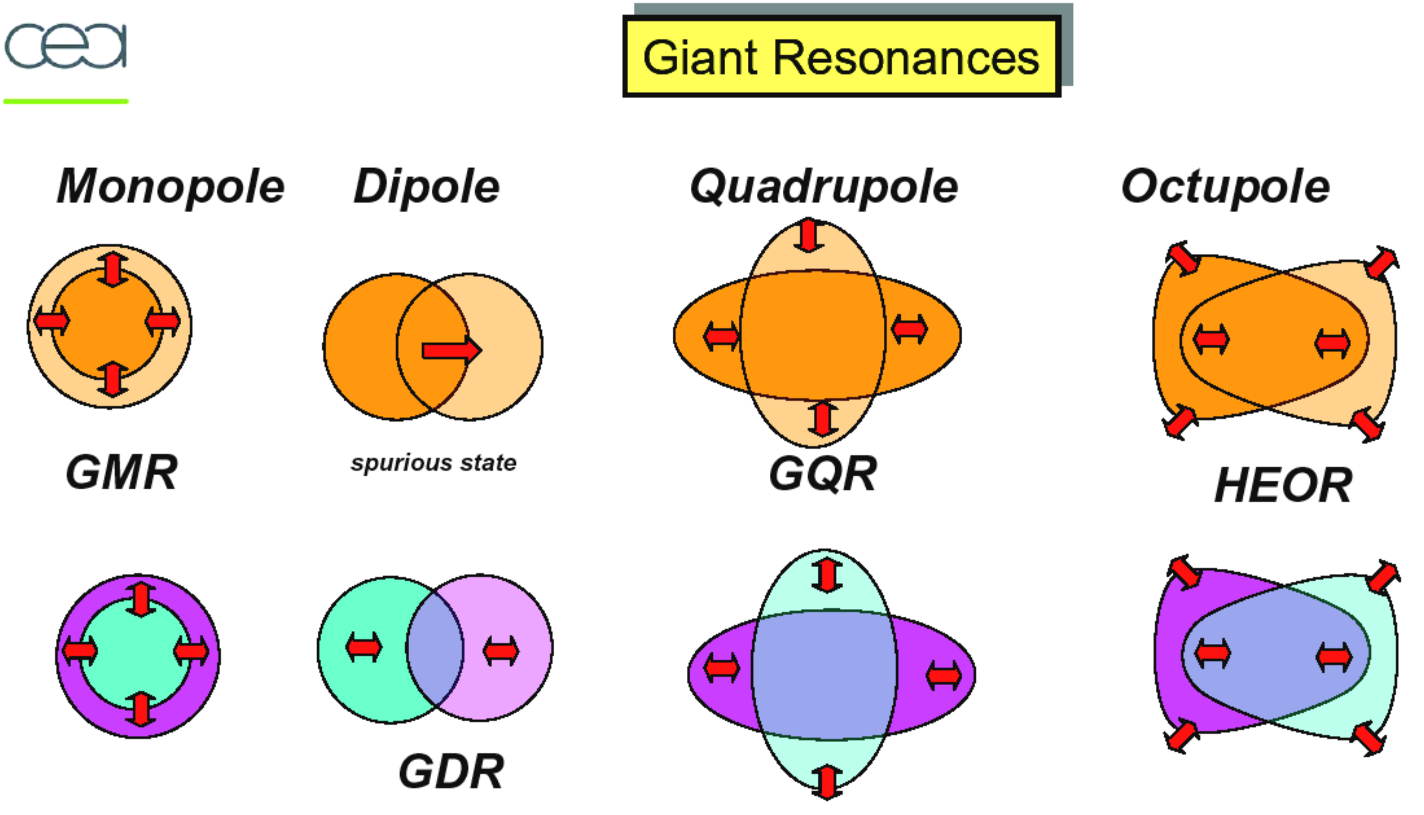




Theory also predicts some exotic shapes for the spinning nucleus

Nuclear Oscillations/Vibrations

A nucleus can vibrate or oscillate in different modes, just like the string of a violin can vibrate with different notes



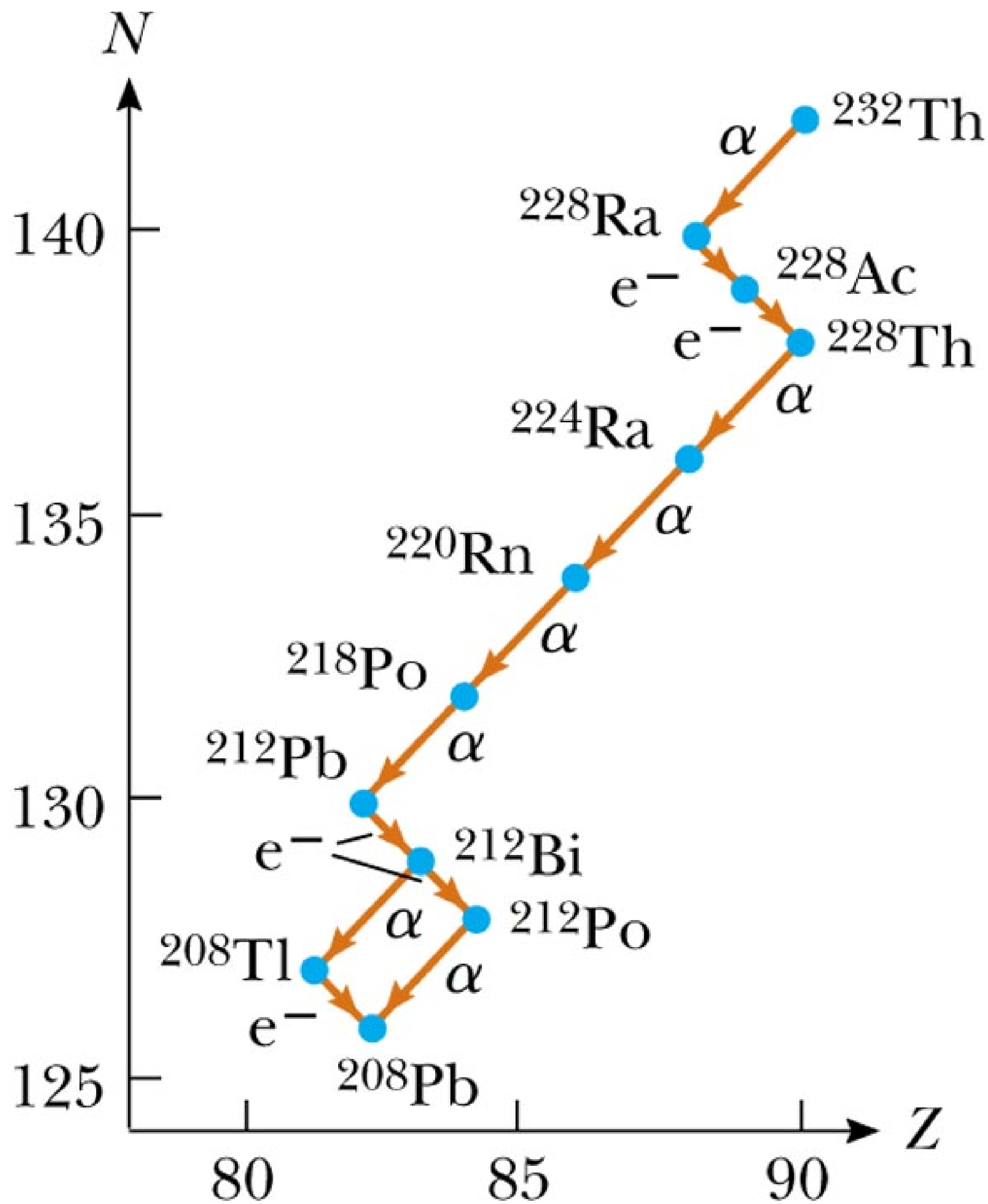
- Protons & neutrons behave as two interpenetrating but separate rigid distributions.
- Rigid distributions undergo harmonic displacement w. r. t each other.
- $E_{GDR} \propto A^{-1/6}$

Classification of Nuclei

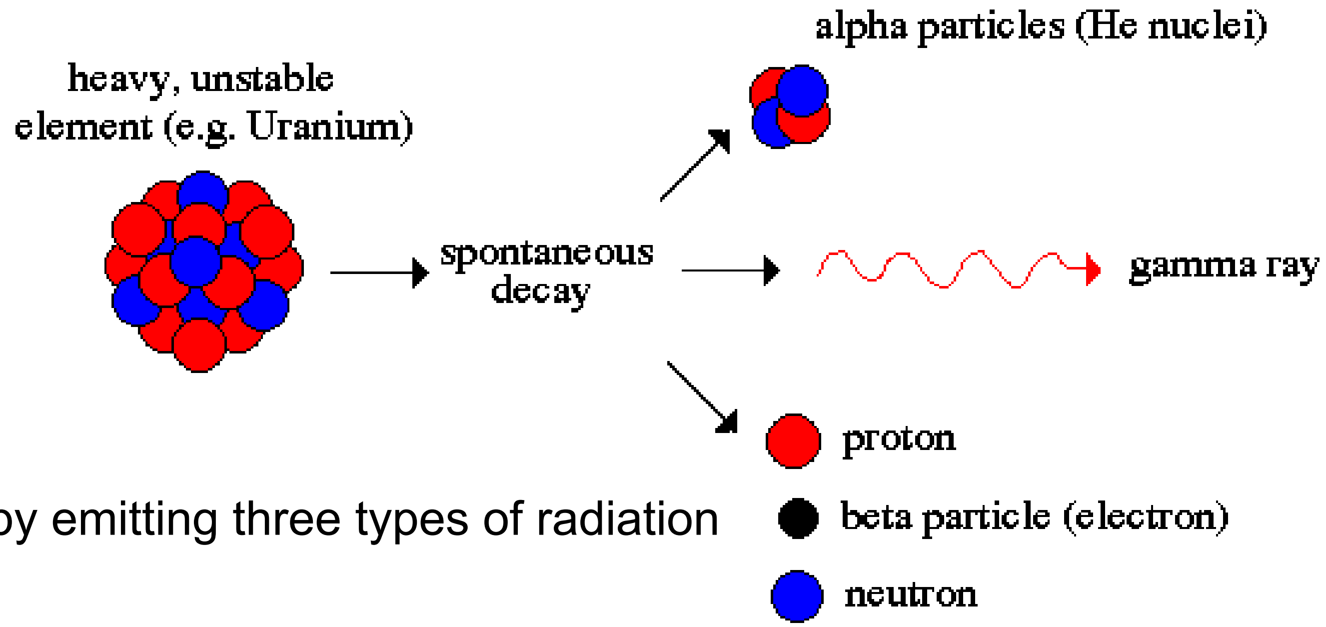
- Classification of nuclei
  - Unstable nuclei found in nature
    - Give rise to *natural radioactivity*
  - Nuclei produced in the laboratory through nuclear reactions
    - Exhibit *artificial radioactivity*
- Three series of natural radioactivity exist
  - Uranium
  - Actinium
  - Thorium

Decay Series of ^{232}Th

- Series starts with ^{232}Th
- Processes through a series of alpha and beta decays
- Ends with a stable isotope of lead, ^{208}Pb



Nuclear Decay



A Nucleus can decay by emitting three types of radiation

- Alpha particles
  - The particles are ^{4}He nuclei
- Beta particles
  - The particles are either electrons or positrons
    - A positron is the *antiparticle* of the electron
    - It is similar to the electron except its charge is +e
- Gamma rays
  - The "rays" are high energy photons

Nuclear Lifetime

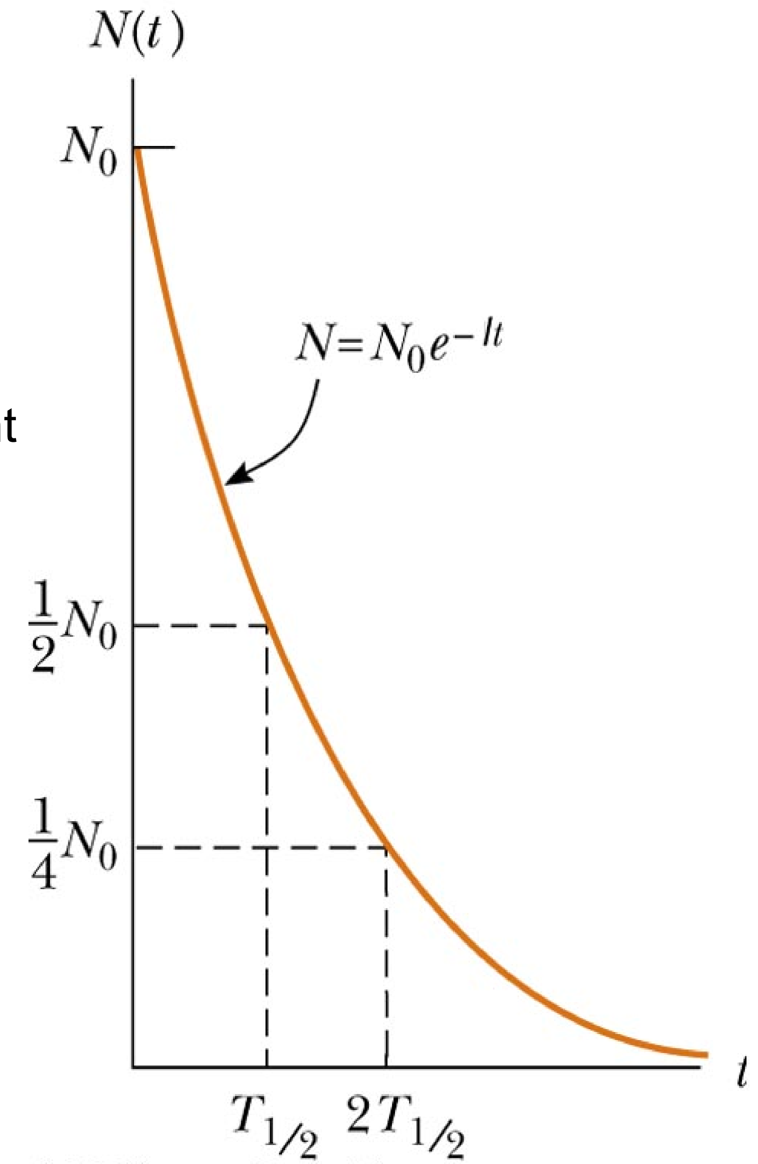
- The number of nuclei that decay in given time follows a decay curve given as

$$N = N_0 e^{-\lambda t}$$

λ – decay constant

- The *half-life* $T_{1/2}$ is also a useful parameter
- The half-life is defined as the time it takes for half of any given number of radioactive nuclei to decay

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

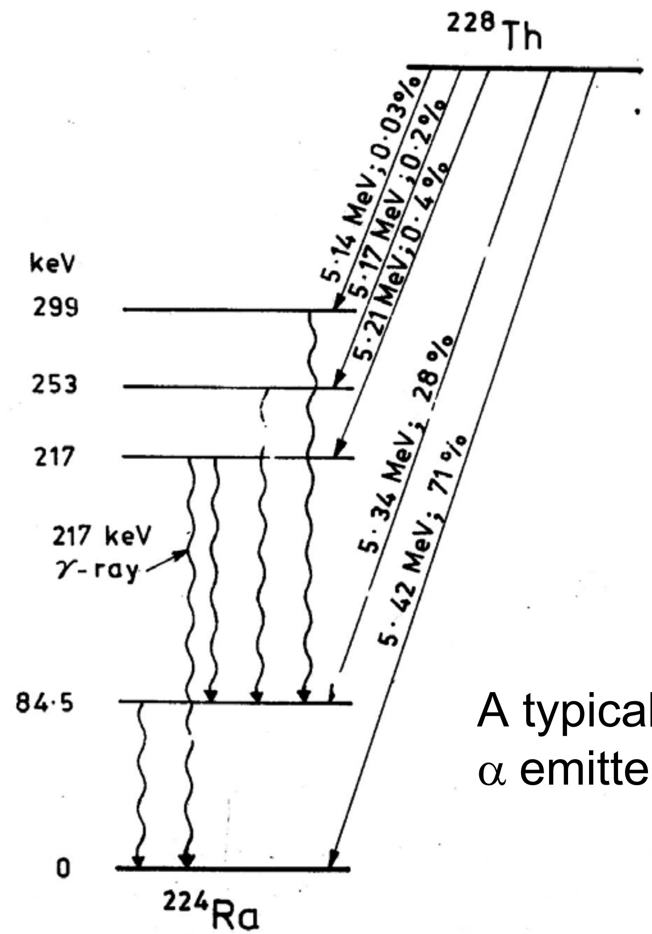


Alpha Decay

- When a nucleus emits an alpha particle it loses two protons and two neutrons
  - N decreases by 2
  - Z decreases by 2
  - A decreases by 4
- Symbolically

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He $$

  - X is called the *parent nucleus*
  - Y is called the *daughter nucleus*



A typical α emitter

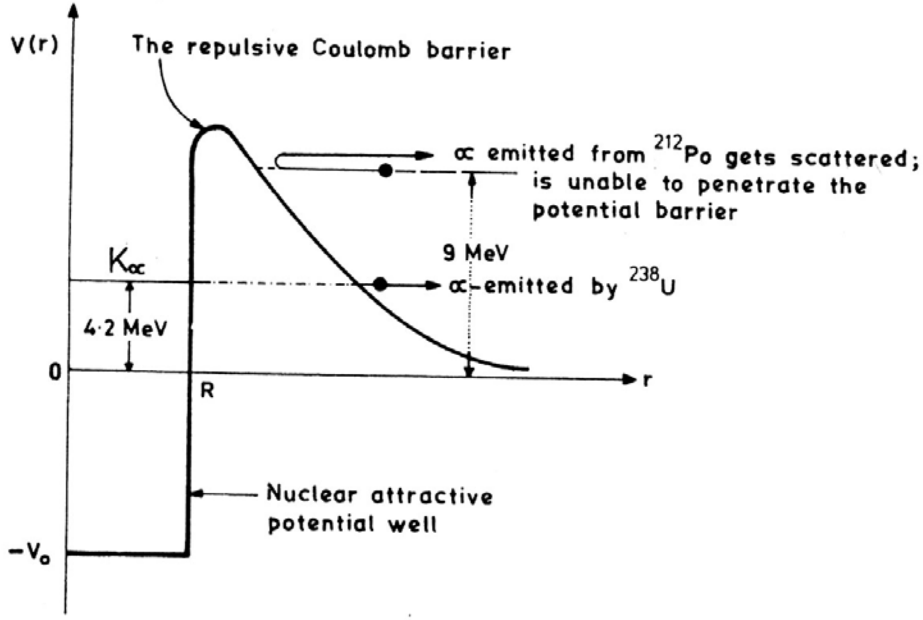
Alpha Decay Paradox

Consider,

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + \alpha \qquad KE(\alpha) = 4.275 \times \left(\frac{234}{238}\right) = 4.2\,MeV$$

A 4.2 MeV a particle is able to come out of the Uranium nucleus

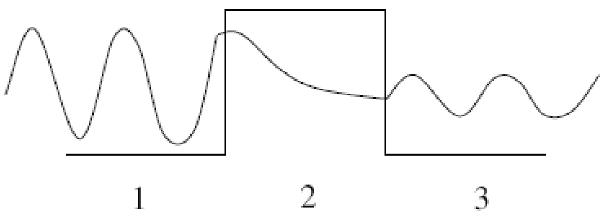
However, α particles with KE(α) = 9 MeV from ^{212}Po are unable to penetrate close enough to $^{238}U_{92}$



If 9 MeV α particle is not able to penetrate the Coulomb barrier from outside, then how is the 4.2 MeV α particle able to penetrate from inside ?

Alpha Decay Paradox – Barrier Penetration

Gamow, Gurney & Condon applied quantum mechanics of particle tunnelling through the barrier to the problem of α decay



$$\phi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$$

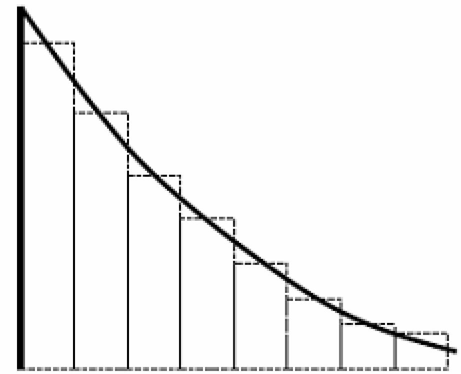
$$\hbar k_1 = \sqrt{2mE}$$

$$\phi_2(x) = Ce^{k_2x} + De^{-k_2x}$$

$$\hbar k_2 = \sqrt{2m(V_0 - E)}$$

$$\phi_3(x) = Fe^{ik_1x}$$

$$T = \frac{F^*F}{A^*A} = \left[1 + \frac{V_0^2}{4E(V_0 - E)}\sinh^2(k_2R)\right]^{-1}$$



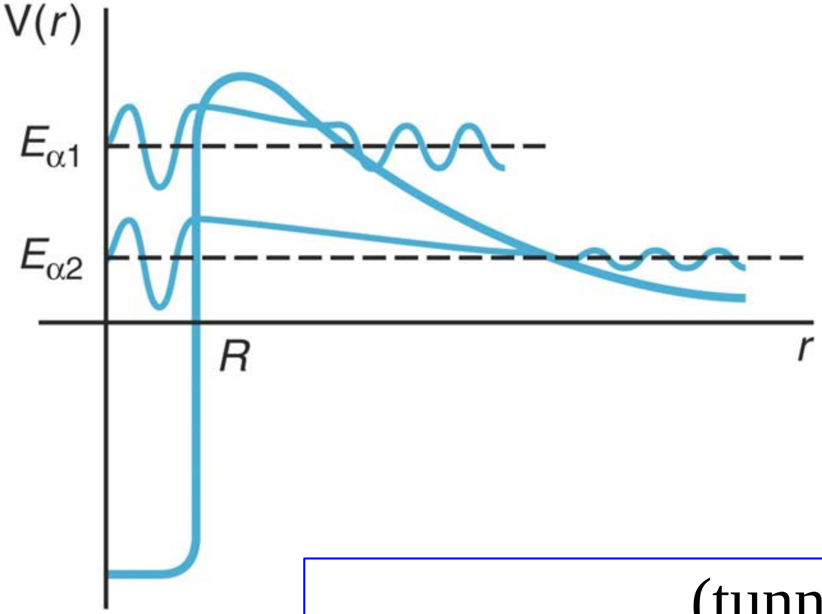
$$T \sim \exp\left\{-a\frac{Z}{\sqrt{E}} + b\sqrt{ZR}\right\}$$

$$a = \frac{e^2\sqrt{2m}}{2\varepsilon_0\hbar} = 3.97\,(MeV)^{1/2}$$

$$b = \frac{8}{\hbar}\sqrt{\frac{me^2}{4\pi\varepsilon_0}} = 2.98\,(fm)^{-1/2}$$

E = α energy in MeV
R = radius of 'daughter' in fm
Z = atomic number of parent

Gamow theory of Alpha Decay



The α particle can tunnel through the potential barrier attempting to confine it to the nuclear interior. The greater the energy the shorter the half-life.

$$\text{decay rate } \lambda = \frac{\text{(tunneling probability)(speed)}}{\text{diameter}} = \frac{Tv}{2R}$$

$$\log t_{1/2} = 1.61\left(ZE_{\alpha}^{-1/2} - Z^{2/3}\right) - 28.9$$

The half-life is in years, the energy is in MeV, and Z refers to the daughter nucleus.

Calculating half-life from the penetration probability T

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + \alpha$$

$$E = 4.2\ MeV, \quad Z_D = 90 \quad \& \quad R \sim 9.3\ fm$$

$$T \sim \exp\left\{-3.97\frac{90}{\sqrt{4.2}} + 2.98\sqrt{90\times 9.3}\right\}$$

$$= \exp(-88) = 6\times 10^{-39}$$

$$6\times 10^{-39}\times 7.5\times 10^{20} = 4.5\times 10^{-18}\ s^{-1}$$

$$= \lambda = \frac{\ln 2}{t_{1/2}}$$

$$\therefore \quad t_{1/2} = 1.54\times 10^{17}\ s = 4.9\times 10^{9}\ yr$$

$$(\text{expt} = 4.46\times 10^{9}\ yr)$$

Time to 'cross' the nucleus is $t = \frac{2R}{v_\alpha}$

Attempt frequency
("knocking rate") is $f = \frac{1}{t} = \frac{v_\alpha}{2R}$

Alpha particle speed = ?

$$E_\alpha \sim 4.2 MeV \qquad (m = 3727.4\ MeV)$$

$$\therefore v_\alpha = \sqrt{\frac{2E}{m}} \sim 1.4\times 10^{7}\ m/s$$

$$\therefore f = \frac{1.4\times 10^{7}}{2\times 9.3\times 10^{-15}} \sim 7.5\times 10^{20}\ s^{-1}$$

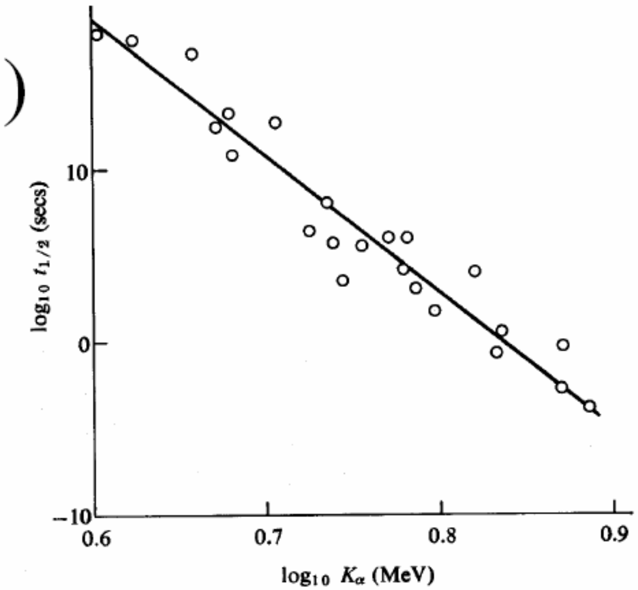
Geiger-Nuttall relation

The α decay theory is able to account for the Geiger-Nuttall law

$$\log t_{1/2} + n \cdot \log K_\alpha = Const.$$

constant

$\log(t_{1/2})$



$\log(K_\alpha)$

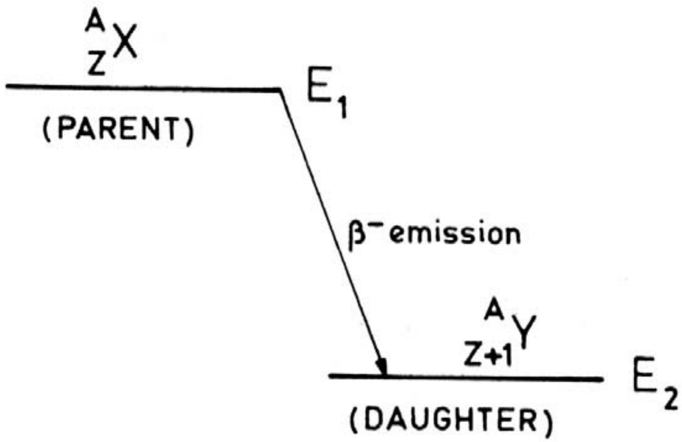
Beta Decay

- Symbolically

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-} + \overline{\nu}$$

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^{+} + \nu$$

  - ν is the symbol for the neutrino
  - $\bar{\nu}$ is the symbol for the antineutrino



- In beta decay, the following pairs of particles are emitted
  - An electron and an antineutrino
  - A positron and a neutrino

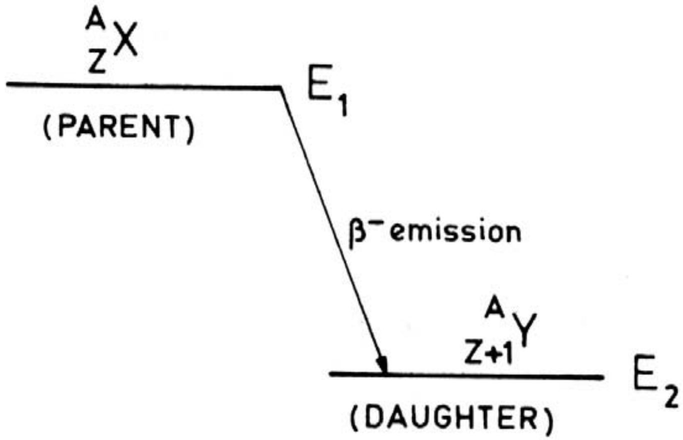
Beta Decay Paradox

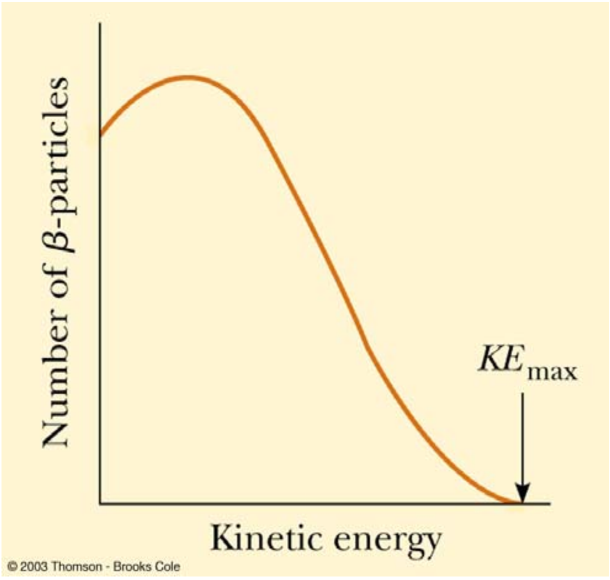
Just like the α decay, β decay also is an energy transition between two definite energy.

Thus, mono-energetic (single energy) β ray is expected.

However, the kinetic energy spectrum of β ray is continuous, implying that the electrons emitted in β decay process have range of kinetic energy.

Also, the beta particle emission violates the conservation of energy and angular momentum.





Pauli's Neutrino Hypothesis

- To account for the continuous energy spectrum and the violation of energy and momentum conservation, Pauli proposed the existence of another particle – the neutrino.

- Pauli postulated that the neutrino must have
  - Zero electrical charge
  - Mass much smaller than the electron, probably not zero
  - Spin of ½
  - And interact very weakly with matter

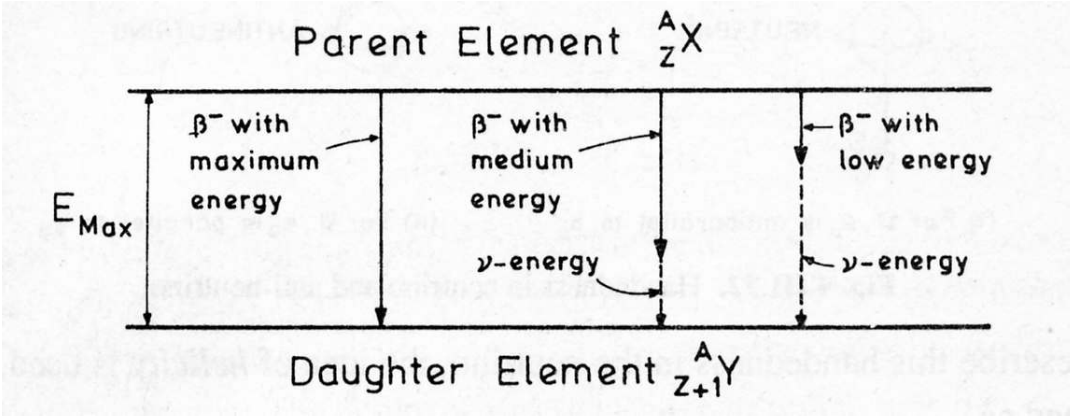


Diagram showing the sharing of total disintegration energy between the b particle and the neutrino

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \overline{\nu}$$

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu$$

Fermi's theory of Beta Decay

Using Pauli's neutrino Fermi proposed a simple theory of β decay using his golden rule

The transition probability is given by

$$\lambda_{fi} = \frac{2\pi}{\hbar} \left| V_{fi} \right|^2 \rho_f$$

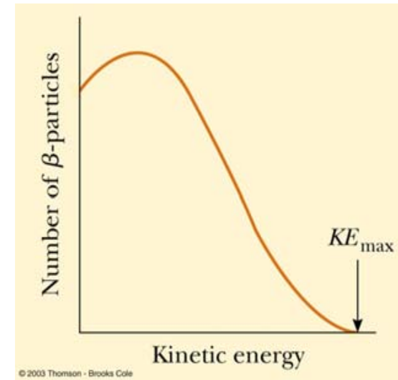
"matrix element" $\quad V_{fi} = \int \psi_f^* V \psi_i \, dv$

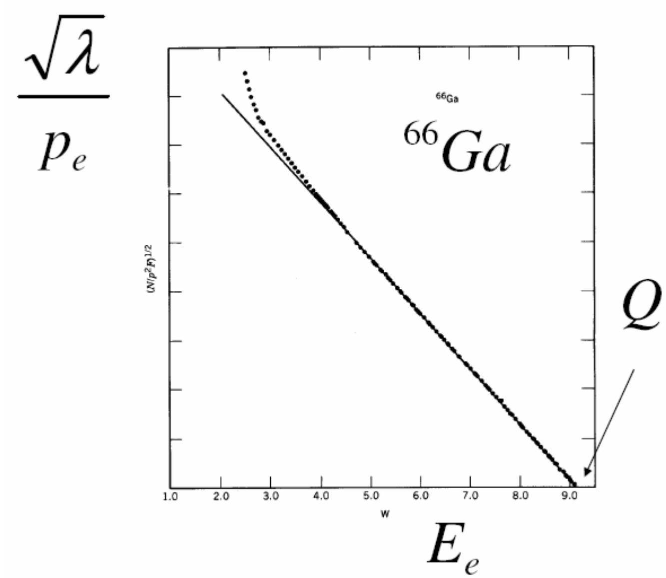
The density of states $\quad \rho \propto p_e^2 (E - E_e)^2$

The transition rate is therefore:

$$\lambda \propto p_e^2 (E - E_e)^2$$

$$\therefore \quad \frac{\sqrt{\lambda}}{p_e} \propto (E - E_e)$$





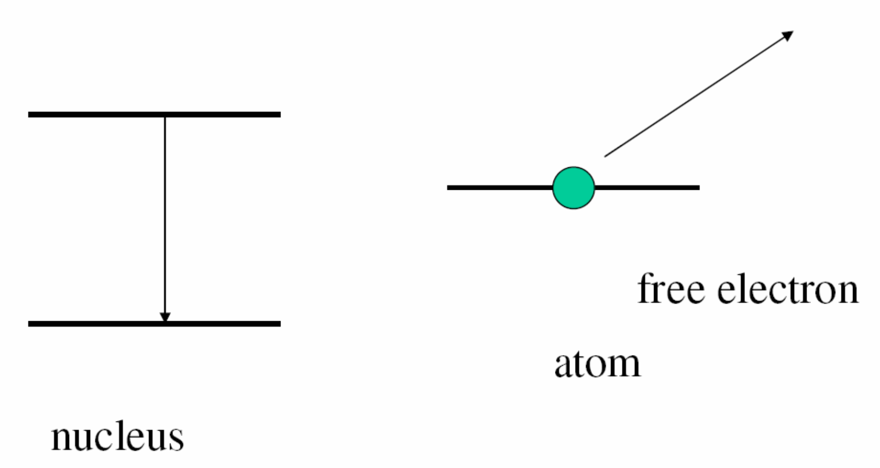
(Fermi-) Kurie plot

Gamma Decay

- Gamma rays are given off when an excited nucleus “falls” to a lower energy state
  - Similar to the process of electron “jumps” to lower energy states and giving off photons
- The excited nuclear states result from “jumps” made by a proton or neutron

$$^{110m}Ag \rightarrow {}^{110}Ag + \gamma$$

- No change in Z, N or A
- Nucleus can also de-excite by ‘Internal Conversion’ (excess energy given to an ATOMIC electron)



Multipolarities in Gamma transition

- Multipole Radiation: Electric and Magnetic
- Opposite parities

$$\pi(EL) = (-1)^L \quad \& \quad \pi(ML) = (-1)^{L+1}$$

- $L = 1 \rightarrow$ Dipole
- $L = 2 \rightarrow$ Quadrupole
- $L = 3 \rightarrow$ Octupole
- $L = 4 \rightarrow$ Hexadecapole etc

- Transition between nuclear states: $I_i \xrightarrow{\gamma} I_f$
- A multipole of order L transfers $L\hbar$ angular momentum per photon

$$\vec{I}_i = \vec{L} + \vec{I}_f$$

$$e.g. \quad (I_i, I_f) = \left(\tfrac{3}{2}, \tfrac{5}{2}\right) \rightarrow L = 1, 2, 3, 4$$

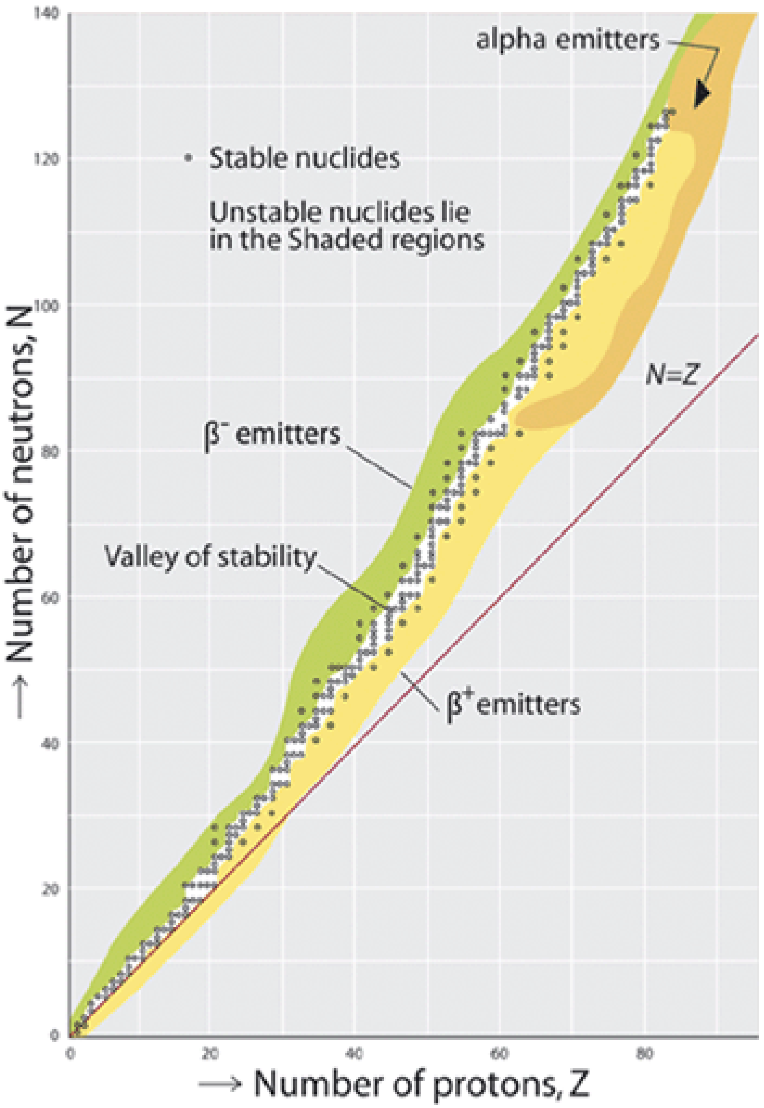
$$i.e. \quad \left|I_i - I_f\right| \leq L \leq \left(I_i + I_f\right)$$

- ‘Electric’ or ‘Magnetic’ depends on parities of nuclear states

Nuclear Stability

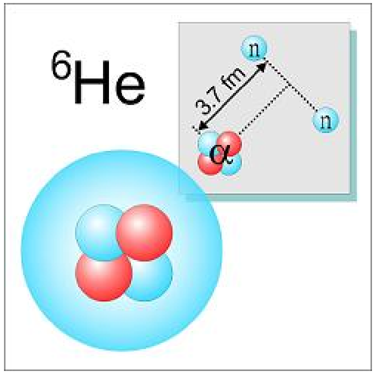
There are 266 stable nuclear isotopes. There are about 3000 radioactive (unstable) nuclides with lifetimes greater than about 1 millisecond.

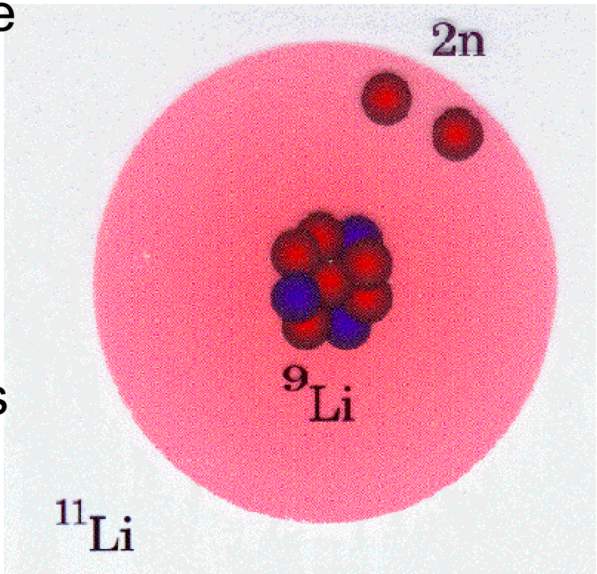
The line of stability lies above the line N=Z because of the Coulomb repulsion between protons.

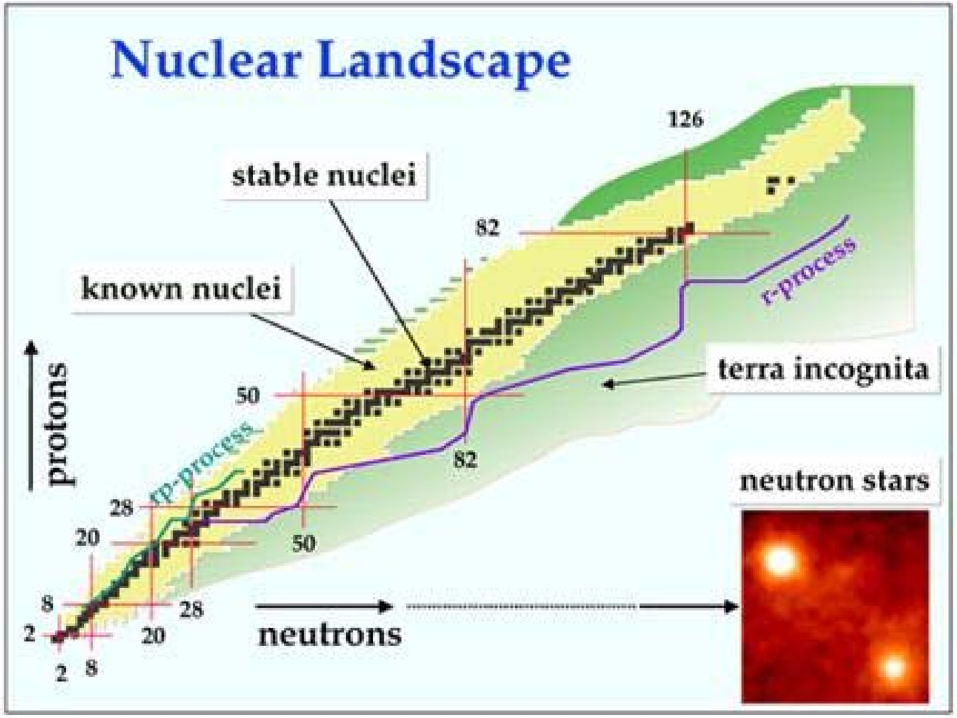


Neutron-rich Nuclei

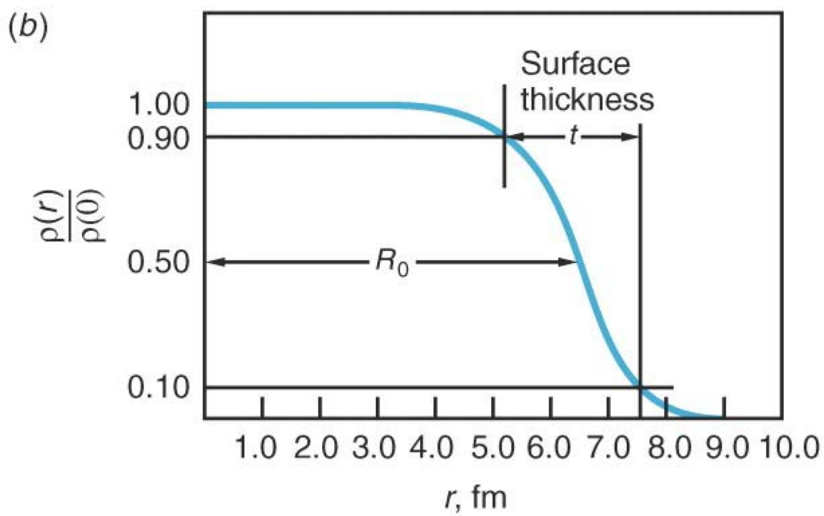
A nucleus can have excess neutrons than those found in stable nucleus and have exotic structures

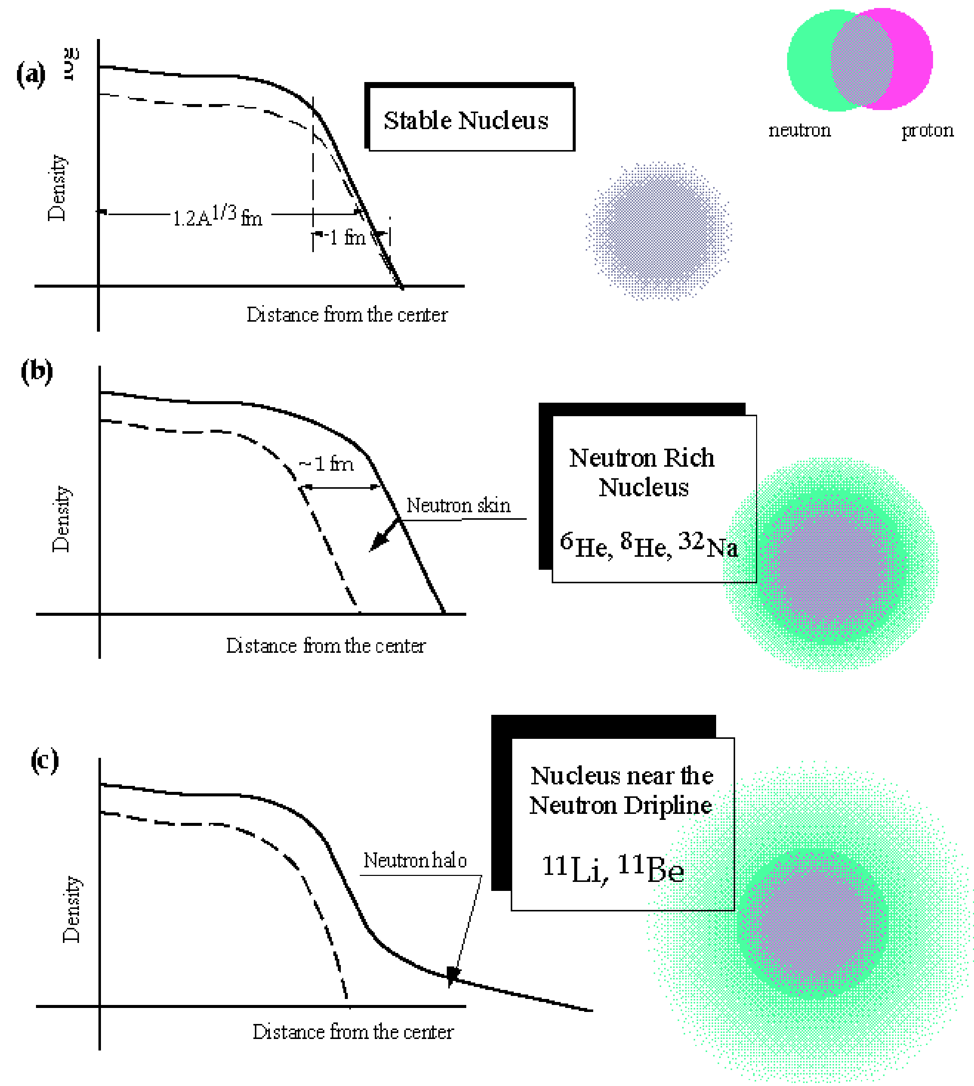






Nuclear Skin Thickness (Halo Nucleus)





The skin thickness t is defined to be the distance from 90% to 10% of the central nuclear density.